Approximate Reasoning in Distributed Systems

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ABSTRACT: In this exposition we sum up our results on approximate reasoning in distributed systems obtained in the last few years. The presented scheme for approximate reasoning is based on rough mereology and encompasses schemes based on fuzzy set theory.

1. INTRODUCTION

We present a formal model of approximate reasoning in distributed systems of intelligent agents. The tasks of considered systems of agents are reduced here to problems of synthesis of a solution to the given requirement.

The accessible knowledge on the basis of which constructs in the synthesis process are selected and classified (evaluated) is as a rule incomplete, poorly defined, or inconsistent. In consequence, we are bound to evaluate the basic ingredients of the synthesis process approximately only, in terms of values of some uncertainty measures which express a degree in which a given construct satisfies a given specification and in terms of some functors which propagate uncertainty measures along the synthesis scheme.

The knowledge of agents is represented via information/decision systems from which the necessary ingredients are extracted. The basic vehicle for carrying out approximate reasoning about objects, approximate negotiations and approximate synthesis of solutions to the tasks (requirements) given to agent systems is the set of similarity measures at individual agents induced by rough mereological inclusions generated from agents knowledge. This approach has been presented, to mention few of our research papers on this theme, in [7-10].

Many formal models of approximate reasoning are described in the literature e.g. Dempster-Schafer theory of evidence [11], bayesian reasoning [6], many-valued logics [1], and fuzzy logics [1], [12].

We can extract from these formal models a general scheme for approximate reasoning.

It is not surprising that this scheme encompasses classical models of reasoning adopted in mathematical logic [4].

2. APPROXIMATE REASONING SCHEME

The scheme for approximate reasoning can be represented by the following tuple

$$Appr\_Reas = (Ag, Link, U, St, Dec\_Sch, O, Inv, Unc\_mes, Unc\_prop)$$

where

(i) The symbol Ag denotes the set of agents (or agent names).

(ii) The symbol $Link$ denotes a finite set of non-empty strings over the alphabet Ag; for $v(ag) = ag_1 ag_2 \ldots ag_k ag \in Link$, we say that $v(ag)$ defines an *elementary synthesis scheme* $synt(ag_1, ag_2, \ldots, ag_k, ag) = synt(v(ag))$ with the *root* ag and the *leaf agents* $ag_1, ag_2, \ldots, ag_k$. The intended meaning of $v(ag)$ is that the agents $ag_1, ag_2, \ldots, ag_k$ are the children of the agent ag which can send to ag some constructs for assembling a complex artifact. The relation $ag \leq ag'$ iff ag is a leaf agent in $synt(v(ag))$ for some $v(ag)$ is usually assumed to be at least an ordering of Ag into a type of acyclic graph; we assume for simplicity that $(Ag, \leq)$ is a tree with the root $root(Ag)$ and leaf agents in the set $Leaf(Ag)$.

(iii) The symbol U denotes the set $\{U(ag) : ag \in Ag\}$ of universes of discourse (universes of constructs) of agents.

(iv) The symbol St denotes the set $\{St(ag) : ag \in Ag\}$ of standard sets of agents: for $ag \in Ag$, the set $St(ag) = \{st(ag)_i\} \subseteq U(ag)$ is the set od *standard constructs* (*objects*) of the agent ag.

(v) The symbol O denotes the set $\{O(ag) : ag \in Ag\}$ of *operations* with $O(ag) = \{o_i(ag)\}$ the set of *operations at* ag.

(vi) The symbol Dec_Sch denotes the set of *decomposition schemes*, a particular decomposition scheme dec_sch_j is a tuple

$$(\{st(ag)_j : ag \in Ag\}, (o_j(ag) : ag \in Ag\})$$

which satisfies the property that if $v(ag) = ag_1 ag_2 \ldots . ag_k ag$ then

$$o_j(ag)(st(ag_1)_j, st(ag_2)_j, \ldots, st(ag_k)_j) = st(ag)_j.$$

The intended meaning of dec_sch_j is that when any child ag_i of ag submits the standard construct $st(ag_i)_j$ then the agent ag assembles from

$$st(ag_1)_j, st(ag_2)_j, \ldots, st(ag_k)_j$$

the standard construct $st(ag)_j$ by means of the operation $o_j(ag)$. The rule dec_sch_j establishes therefore a decomposition scheme of any standard construct at the agent $root\ (Ag)$ into a set of consecutively simpler standards at all other agents. The standard constructs of leaf agents are primitive standards. We can regard the set of decomposition schemes as a skeleton about which the approximate reasoning is organized. Any rule dec_sch_j conveys a certain knowledge that standard constructs are synthesized from specified simpler standard constructs by means of specified operations. This ideal knowledge is a reference point for real synthesis processes in which we deal as a rule with constructs which are not standard: in adaptive tasks, for instance, we process new, unseen yet, constructs (objects, signals).

(vii) The symbol Inv denotes the *inventory set* of primitive constructs.

(viii) The symbol Unc_mes denotes the set $\{Unc\_mes(ag) : ag \in Ag\}$ of *uncertainty measures of agents*, where $Unc\_mes(ag) = \{\mu_j(ag)\}$ and $\mu_j(ag) \subseteq U(ag) \times U(ag) \times V(ag)$ is a relation (possibly function) which determines a distance between constructs in $U(ag)$ valued in a set $V(ag)$; usually, $V(ag) = [0, 1]$, the unit interval.

(ix) The symbol Unc_prop denotes the set of *uncertainty propagation rules* $\{Unc\_prop(v(ag)) : v(ag) \in Link\}$; for $v(ag) = ag_1 ag_2 \ldots . ag_k ag \in Link$, we have in $Unc\_prop(v(ag))$ the functions $f_j : V(ag_1) \times V(ag_2) \times \ldots . \times V(ag_k) \longrightarrow V(ag)$ such that

if $\mu_j(ag_i)(x_i, st(ag_i)_j) = \varepsilon_i$ for $i = 1, 2, \ldots, k$

$$\text{then } \mu_j(ag)(o_j(x_1, x_2, \ldots, x_k), st(ag)_j) = \varepsilon \geq f_j(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k).$$

The functions f_j propagate uncertainty measures from children of ag to ag. The process of synthesis begins at leaf agents which receive primitive constructs and calculate their distances from their respective standards; then the primitive constructs are sent to the parent nodes of leaf agents along with vectors of distance values. The parent nodes synthesize complex constructs from the sent primitives and apply the uncertainty propagating functions in order to calculate from the sent vectors the new vectors of distances from their respective standards. Finally, the root agent $root(Ag)$ receives the constructs from its children from which it assembles the final construct and calculates the distances of this construct from the root standards. On the basis of the found values, the root agent classifies the final construct.

Our approach is anchored in rough set theory [5].

The formal treatment of partial containment is provided by the notion of a rough inclusion [7-10]. Rough inclusions are construed as most general functional objects conveying the intuitive meaning of the relation of being a part in a degree. In particular, the relation of being a part in the greatest possible degree is the relation of being a (possibly, improper) part in the sense of mereology of Stanislaw Leśniewski [2]. We can regard therefore a rough inclusion as a measure of departing from a decomposition scheme represented by the induced model of mereology of Leśniewski.

In mereology of Leśniewski the notions of a subset and of an element are equivalent and therefore we can interpret rough inclusions as global fuzzy membership functions on the universe of discourse which satisfy certain general requirements responsible for their regular mathematical properties.

3. THE SYNTHESIS PROCESSES

The process of synthesis of a complex system by a scheme of agents consists in our approach of the two communication stages viz. the top - down communication/negotiation process and the bottom - up communication process. We outline the two stages here.

In the process of top - down communication, a requirement Φ received by the scheme from an external source is decomposed into approximate specifications of the form

$$(\Phi(ag), \varepsilon(ag))$$

for any agent ag of the scheme. The intended meaning of the approximate specification $(\Phi(ag), \varepsilon(ag))$ is that a construct $x \in U(ag)$ satisfies $(\Phi(ag), \varepsilon(ag))$ iff there exists a standard $st(ag)$ with the properties that $st(ag)$ satisfies the predicate $\Phi(ag)$ and

$$\mu(ag)(x, st(ag)) \geq \varepsilon(ag).$$

The uncertainty bounds of the form $\varepsilon(ag)$ are defined by the agents viz. the root agent *root(Ag)* chooses $\varepsilon(root(Ag))$ and $\Phi(root(Ag))$ as such that according to it any construct x satisfying $(\Phi(root(Ag), \varepsilon(root(Ag))$ should satisfy the external requirement Φ in an acceptable degree; the other agents choose their approximate specifications in negotiations within each elementary scheme $synt(v(ag))$ for $v(ag) \in Link$. The result of the negotiations is succesful when there exists a decomposition scheme dec_sch_j such that for any $v(ag) \in Link$, where $v(ag) = ag_1 ag_2 \ldots ag_k ag$, from the conditions $\mu(ag_i)(x_i, st(ag_i)_j) \geq \varepsilon(ag_i)$ and $st(ag_i)_j$ satisfies $\Phi(ag_i)$ for $i = 1, 2, \ldots, k$, it follows that $\mu(ag)(o_j(x_1 x_2, \ldots, x_k), st(ag)_j) \geq \varepsilon(ag)$ and $\mathrm{st(ag)}_j$ satisfies $\Phi(ag)$.

The uncertainty bounds $\varepsilon(ag)$ are evaluated on the basis of uncertainty propagating functions whose approximations are extracted from information systems of agents.

Any leaf agent realizes its approximate specification by choosing in the subset $Inv \cap U(ag)$ of the inventory of primitive constructs a construct satisfying this specification.

The bottom-up communication consists of agents sending to their parents the chosen constructs and vectors of their rough mereological distances from the standards. The root agent $root(Ag)$ assembles the final construct.

Our approach is analytic in the sense that all objects necessary for the synthesis process are extracted from the empirical knowledge of agents represented in their information systems; it is also intensional in the sense that rules for propagating uncertainty are local as they depend on a particular elementary synthesis scheme and on a particular local standard.

In our presentation, we will outline basic notions of the rough set theory and mereology of Leśniewski, rough mereology and a more detailed analysis of algorithms for approximate synthesis of complex objects on the basis of knowledge encoded in information systems of agents.

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